Mathematical Analysis of Two-Phase Flow in Low-Permeability Porous Media

A one-dimensional isothermal transient two-phase flow model of gas and liquid in a low-permeability medium has been developed from the general mass and momentum balances.

The characteristic roots of the governing differential equations are real and distinct. Therefore, solution of the flow equations may be attempted by a stable initial-value finite-difference scheme. A numerical solution was obtained using the method of lines.

Calculated two-phase flow parameters are consistent with expected flow behavior and reflect the sensitivity of fluid flow to porosity, gas and liquid permeability, and capillary pressure in a tight sands porous medium. Calculated gas flow rates compared well with experimental data obtained at the Institute of Gas Technology (IGT) for low-permeability sandstone.

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Introduction

During the past decade, there has been increasing attention on economical gas recovery from unconventional reservoirs, particularly low-permeability (tight sand) formation. Sharer and O'Shea (1986) estimated over 14×10^{12} μm^3 of natural gas may be recoverable from low permeability sandstone located in the United States. To obtain feasible production capability, significant research and development, both experimental and theoretical, is needed to develop reliable experimental data and mathematical description of two-phase flow as well as to advance massive hydraulic fracturing technology (MHF). Gas bearing, tight (low permeability) reservoirs could be present in sandstone, siltstone, silty shale, and chalk. They usually have an *in-situ* permeability to gas of 0.1 md (987×10^{-7} μm^2) or less. Tight gas reserves exhibit several characteristics that are unique when compared to conventional reservoirs. Among the most significant is the relationship between porosity and permeability which is reasonably consistent for conventional reservoirs, but completely inconsistent for tight gas reservoirs.

The uniqueness of naturally-occurring low-permeability materials reduces the reliability of simulation techniques applied to conventional porous media. In higher-permeability media, application of Darcy's Law, which is not appropriate in some cases, is standard practice under single-phase flow conditions. When more than one flowing phase is present, Darcy's Law is usually applied to each phase with modification of the permeability terms. Relative permeability is incorporated into the relations to account for the deviation from single-phase flow behavior. Typically relative permeability is expressed as a nonlinear function which is directly proportional to phase saturation. The effective permeability of a fluid approaches its single-phase permeability, as its saturation in the medium increases.

In most of gas and liquid flow in high-permeability porous medium, both phases are in intimate contact throughout the medium. On a pore level, both phases are moving in the same space via some mechanism such as annular flow, slug flow, and stratified flow, and the Darcy's law has been modified in terms of relative velocity between the phases. We believe, in the low-permeability media, surface tension effects can cause the segregation of fluids inside the media. Thus, both gas and liquid do not coexist in the individual pores.

The literature contains several fundamental studies on the experimental flow behavior in low permeability media, such as Jones and Owens (1980), Walls et al. (1982), Randolph (1983), Freeman and Bush (1983), Chowdiah (1986), and Soeder (1986). Most of these studies provide us with a better understanding of tight-sand-media gas permeability in the presence of water and confining stress. There have been, however, few attempts in theoretical analysis of only single-phase flow through dry or partially-saturated low-permeability media. Arastoopour and Adewumi (1983) obtained analytical solution to the linearized gas flow equation in porous media. Newberg and Arastoopour (1986) developed continuity and momentum balances for flow of gas through low-permeability media and successfully compared their calculated values with IGT's experimental data (Randolph, 1983; Chowdiah and Arastoopour, 1983).

This study is the first attempt in formulation and solution of two-phase flow through low-permeability media. The approach we have taken to model multiphase flow in low-permeability media is the application of the general mass and momentum balances to segregated mobile fluid phases. Single-phase permeabilities and fluid volume fractions are used in determining interfacial friction forces. Furthermore, a condition of capillary equilibrium was assumed in the medium.

The resulting model has capability of predicting transient and steady-state single- and two-phase flow in low-permeability media.

Physics of Multiphase Flow in Porous Media

A fluid's resistance to flow is a result of viscous dissipation. The fluid's motion is a function of the properties of the fluid, the medium through which it is travelling, and the imposed external forces.

Consider flow in a naturally-occurring consolidated porous medium. A porous medium is often viewed as a matrix of randomly-connected flow passageways of various dimensions. Then, a significant adjustment to pipe flow is required because of the complexity of the flow passages. Macroscopic view is usually taken instead of trying to model each pore (flow passage) on an individual basis. Thus, overall parameters are used to represent the average motion of fluid through the medium. One such parameter is porosity. Two types of porosity are introduced to represent the void space. The total porosity, γ, is the fraction of the total effective void space per unit volume of the medium, while the flow porosity, ϕ, is the void space available for fluid flow per unit volume of the medium. The difference is the pores which are not part of a continuous path through the medium. The frictional forces in porous media are expressed in terms of permeability. The higher the permeability, the easier it is for the fluid to pass through the medium (i.e., a lower pressure drop is required). Quantifying permeability is more complex than the use of Reynolds Number and surface roughness. Many attempts have been made to determine the properties of the fluid and solid which influence permeability, such as tortuosity of pores and porosity. For our discussions, we will not attempt to define the form of this function. We will only assume it is an experimentally-determined characteristic of the fluid/medium combination.

The one-dimensional steady isothermal nonreacting single-phase flow in a homogeneous porous medium, at constant velocity, may be approximated by Darcy's Law:

$$\frac{\partial P_l}{\partial x} = \frac{-\mu_l \phi}{k_l} U_l \qquad (1)$$

where k_l is liquid permeability, P_l is liquid pressure, U_l is liquid velocity, and μ_l is liquid viscosity.

Next, we introduce a second immiscible fluid, say a gas, into the medium. Now we have a gas/liquid interface as well as another fluid/matrix interface. Also, we now have to consider how the driving force, pressure drop across the medium, is to be distributed in the mobile phases.

The presence of more than one mobile phase necessitates the inclusion of the volume fraction of each fluid. A fluid's volume fraction is that volume of the available void space occupied by a fluid divided by the total available void space. For our two-phase system, the volume fractions are related:

$$S_l + S_g = 1 \qquad (2)$$

where S_l and S_g are the liquid and gas volume fractions.

The effective permeability of a fluid in a medium is that fluid's absolute permeability (that fluid's single-phase flow permeability measured when it totally saturates the medium), corrected by a relative permeability term. Relative permeability is an experimentally-determined function of fluid saturation which approaches unity as the fluid totally saturates the medium. There are published expressions for relative permeability, which have proven useful in certain fluid/medium applications (Corey, 1954).

The existence of a gas/liquid interface introduces a local pressure discontinuity between the phases. For example, when gas and water are in contact with each other inside the pores, water will tend to coat the surface, which is referred to as wetting fluid. Similarly, the gas phase is referred to as nonwetting fluid. The difference between fluid pressures on opposite sides of a gas/liquid interface is called capillary pressure:

$$P_c = P_g - P_l \qquad (3)$$

where P_g is the pressure in the nonwetting gas phase, P_l is the wetting liquid pressure, and P_c is the capillary pressure. The magnitude of the capillary pressure is related to the curvature of the interface between fluids. The smaller the radius of curvature, the greater is the difference between fluid pressures (Slattery, 1970; Collins and Cook, 1959; Glasstone, 1962).

Darcy's relation has been extended directly to each phase in conventional reservoirs. The conventional reservoirs have the porosity which ranges from 8% to 38% and permeabilities from 1 md ($987 \times 10^{-6}\ \mu m^2$) up to 1 d ($987 \times 10^{-3}\ \mu m^2$). Since capillary pressure is very low, it is typically neglected in the fluid flow calculations.

In contrast, low-permeability media have porosities from 2.5% to 12%, permeabilities on the order of 1 md ($987 \times 10^{-6}\ \mu m^2$) or less, and capillary pressures ranging from tens of kPa to several thousand kPa depending on wetting phase content. This high capillary pressure is a consequence of the small characteristic dimensions of the pores. We have assumed that a state of capillary equilibrium is appropriate for these systems even under flow conditions. The capillary number relates the magnitude of viscous and capillary forces:

$$\text{Capillary Number} = \frac{\mu U}{\sigma} \qquad (4)$$

where μ is viscosity, U is velocity, and σ is surface tension. The capillary number for gas and water in a low-permeability porous medium is on the order of 10^{-5}. This indicates that the viscous driving forces are small in comparison with those of capillary pressure.

Rapoport and Leas (1951) found it is reasonable to assume capillary equilibrium for flow conditions prevailing throughout most of a gas drive reservoir containing oil and gas. Also, in reviews of two-phase flow phenomena in porous media conducted by Wooding and Morel-Seytoux (1976) and Philip (1972), they suggested that, if the capillary number is less than

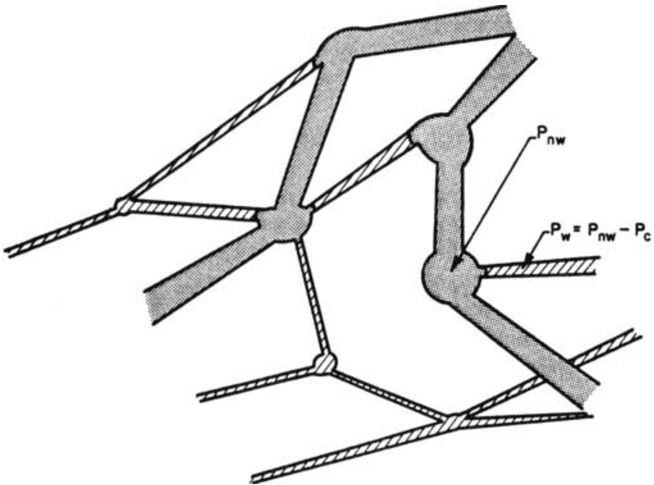

Figure 1. Conception of two-phase flow-through low-permeability media.

one, the shape of the fluid/fluid interface should be independent of velocity.

A state of capillary equilibrium would cause the formation of discrete gas/liquid interfaces as shown in Figure 1. The mathematical consequence of this physical condition is the justification of the interfacial drag terms used, which is based on phase velocities, rather than the relative velocity. Whitaker (1985) used intuition to justify this concept of neglecting relative velocity terms. He concluded that, as long as the difference in the viscosity of the fluids is great, the fluids will exert negligible viscous stress on each other. Furthermore, Philip (1972) found the momentum transfer between fluids in a porous medium to be insignificant based on a survey of the few case studies. Thus, due to a very low capillary number in low permeability media, we have assumed that the friction felt by a particular flowing phase should be a function of that fluid's physical properties, volume fraction, velocity, and the porous medium properties, not the velocity of other mobile phases. Our view is that the flow of gas or liquid phase in the medium goes through a network of pores of various sizes interconnecting at a sufficient number of points so that the pressure in each mobile phase can be considered uniform at any cross section. We also assume that there is a segregation of mobile phases in a partially saturated medium. The wetting fluid, liquid in our system, would occupy the smaller pores; nonwetting fluid occupies larger voids. As a result of capillary equilibrium, there would not be a continuous interface of mobile wetting and mobile nonwetting fluids. The mobile fluids would have interfaces only at nodes where pores full of wetting fluid intersect capillaries containing nonwetting fluids and in pores where plug flow behavior is occurring.

This concept of flow behavior on a pore level also allows the assumption that a gradient in the pressure of one mobile phase will affect the fluid pressure of another mobile phase in the medium such that at any cross section their pressure can be related to Eq. 3. For example, consider the injection of gas into a low permeability sandstone partially saturated with water. A gradient in the gas pressure across the sample will affect the liquid pressure at points where the two phases share an interface. The wetting-phase pressure, P_w, at these interfaces is equal to the nonwetting-phase pressure, P_{nw}, minus the capillary pressure, P_c, at that point. This causes a pressure gradient in the liquid phase. Even though the liquid is flowing in a plug flow fashion in individual pores, on a macroscopic volume-averaged basis, we will treat this mechanism as the simultaneous flow of two continuous phases through the porous medium.

Mathematical Representation of the Model

The mathematical representation of our model for the transient isothermal one-dimensional two-phase flow of gas and liquid through a nondeformable porous solid matrix may be derived from volume-averaged equation for two-phase flow in porous media (Whitaker, 1973, 1985; Gray and O'Neil, 1976; Slattery, 1967). The basic assumptions made in the development of our governing relations are:

1. The system consists of two immiscible, nonreacting fluids in a nondeformable porous medium.
2. Each phase can be described by the equations of continuum mechanics.
3. Interfacial drag terms can be used to represent viscous dissipation effects.
4. Pressure drop in both phases is coupled with the condition of capillary equilibrium.

We allowed for the distinction of mobile and immobile phases of the fluids present as:

S_g = total gas volume fraction

S_{gl} = immobile gas volume fraction

S_l = total liquid volume fraction

S_{ll} = immobile liquid volume fraction

This enables us to exclude portions of the fluids from contributing to the flow of their respective phases. For example, it might be assumed that the irreducible liquid volume fraction is all trapped in dead end pores and not available for flow even at a high liquid pressure gradient. We also have assumed that the volume fraction of immobile phases are not functions of time but can be position-dependent.

Using the groupings of variables:

$$A = \phi(l - S_l - S_{gl})\,\rho_g \tag{5}$$

$$B = \phi(l - S_l - S_{gl})\,\rho_g U_g \tag{6}$$

$$E = \phi(S_l - S_{ll})\,\rho_l \tag{7}$$

$$F = \phi(S_l - S_{ll})\,\rho_l U_l \tag{8}$$

$$F_{eg} = \frac{-\mu_g \phi^2\,(l - S_l - S_{gl})^2}{k_g} U_g \tag{9}$$

$$F_{el} = \frac{-\mu_l \phi^2\,(S_l - S_{ll})^2}{k_l} U_l \tag{10}$$

The resulting conservative form of the partial differential equations are:

Conservation of mass:

Gas Phase:

$$\frac{\partial A}{\partial t} + \frac{\partial B}{\partial x} = 0 \tag{11}$$

Liquid Phase:

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{12}$$

Conservation of momentum:

Gas phase:

$$\frac{\partial B}{\partial t} + \frac{\partial (B^2/A)}{\partial x} = -\phi(l - S_l - S_{gl})\frac{\partial P_g}{\partial x} + F_{eg} \tag{13}$$

Liquid phase:

$$\frac{\partial F}{\partial t} + \frac{\partial (F^2/E)}{\partial x} = -\phi(S_l - S_{ll})\frac{\partial P_l}{\partial x} + F_{el} \tag{14}$$

Capillary pressure equation:

$$P_g - P_l = P_c(S_l) \tag{15}$$

It has been assumed that the flow porosity and volume porosity are equal and constant. F_{eg} and F_{el} are gas and liquid single-phase friction forces. $P_c(S_l)$ is the capillary pressure of the gas-liquid system in the porous medium, which is assumed to be a function of saturation.

Analysis of Characteristics

Our system of equations is nonlinear. It is desirable to solve the system of PDE's numerically as an initial-value problem. In this approach, the system starts at a known condition at time zero. Then a change of boundary conditions is introduced at prescribed locations. The propagation of these disturbances through the medium as a function of time is then evaluated. To determine if our numerical technique which marches in time is stable, it is necessary to know whether the mathematical nature of our equations is well posed as the initial value problem or not.

The mathematical interpretation of "well posedness" may be described by the characteristic roots of the system of differential equations. These roots are the vectors that define the direction of propagation of a disturbance in the physical system. Once they are found, these vectors indicate what type of equations we have (i.e., hyperbolic, parabolic, or elliptic), thereby showing if the problem can be solved on an open or closed region for the boundary/initial value problems. If the system of equations possess as complex-valued characteristics, it is not well posed as an initial value problem (Lax, 1958). It is worth mentioning that real characteristics are not sufficient conditions to guarantee a well-posed problem, since proper initial and boundary data also need to be prescribed.

Lyczkowski et al. (1978) showed that the ill posedness of initial valued problems for the two-phase flow partial differential equations that posses complex characteristics produces unstable numerical schemes. However, if the growth rate resulting from the complex characteristics remains small throughout the time span of the numerical experiment, even the unstable numerical schemes can produce apparently stable results. Furthermore, sufficient numerical damping for the assigned increment size may also result in stable solutions.

In our two-phase flow-through porous media equations, we considered the pressure drop in both gas and liquid phases. In spite of disagreements among the researchers in handling the pressure gradient terms to make the equations realistic and well posed, our equations appear to be well posed as initial valued problem because of different gas and liquid pressure in our two phase flow equations. Substituting liquid pressure in terms of the gas pressure and capillary pressure, which is a function of saturation, results in additional interfacial terms $[\phi(S_l - S_{ll})\,(\partial P_c(S_l))/(\partial x)]$ besides gradient of gas pressure in the liquid momentum equation. This term may be written in terms of gradient of saturation:

$$\phi(S_l - S_{ll})\frac{\partial P_c(S_l)}{\partial x} = \phi(S_l - S_{ll})\frac{\partial P_c(S_l)}{\partial S_l}\frac{\partial S_l}{\partial x} = F(S_l)\frac{\partial S_l}{\partial x} \tag{16}$$

The above interfacial force is similar to solid stress terms which were introduced by Rietma and Mutsers (1973) and used by Gidaspow et al. (1983) for describing gas-solids flow behavior in a fluidized-bed system. For two-phase flow-through high-permeability media, the interfacial term which is the result of capillary pressure, makes the system numerically stable, but does not contribute significantly to the numerical values of the flow parameters. In two-phase flow-through low-permeability media, the interfacial term due to capillary effect not only makes the system numerically stable, but also play a significant role in predicting numerical values for the flow parameters.

Characteristic curves

Our two-phase-flow equations are a set of first-order partial differential equations with two independent variables, *x* (flow direction), and *t* (time). Our equations may be written in a matrix form as:

$$A\frac{\partial U}{\partial t} + B\frac{\partial U}{\partial x} = C \tag{17}$$

where *A* and *B* are (4 x 4) matrices, and *U* and *C* are column matrices. The characteristics were calculated using Eq. 18.

$$|A\lambda + B| = 0 \tag{18}$$

Because of the complexity of Eq. 18, we could not obtain analytical expression for characteristics. Thus, we developed a computer program to solve the equation numerically at the low-permeability gas reservoir conditions. Furthermore, the following constitutes equations or additional assumptions that were made:

1. Capillary pressure was assumed to be an exponential function of liquid volume fraction fitted to experimental data.
2. The gas density relation was taken as:

$$\rho_g = \frac{P_g MW}{zRT} \tag{19}$$

where *MW* is the gas molecular weight, *R* is the ideal gas constant, T is temperature, and z is the gas compressibility factor and is calculated using the Peng-Robinson equation of state.

3. The liquid density is given by:

$$\rho_l = \rho_{lo} \exp C(P_l - P_{lo}) \tag{20}$$

where ρ_{lo} is the liquid reference density at reference pressure P_{lo}, and C is the liquid compressibility factor.

The characteristic roots were determined for three cases:

1. Both phases incompressible
2. An incompressible liquid phase, but a compressible gas phase
3. Both phases compressible

The variables were assigned as follows:

- Pressures of 4×10^6 to 7×10^7 N/m^2
- Temperature of 30 to 150°C
- Porosities of 0.025 to 0.4
- Pore gas velocities of 2×10^{-3} to 2×10^{-5} cm/s
- Liquid pore volume fraction 0.1 to 0.7

The roots were found to be real and distinct for all conditions tested. Thus, the solution of this system of equations may be propagated in an open region along the characteristics using a stable initial-value finite-difference scheme, with the character of the initial profile being conveyed by the eigenvalues and impressed upon successive profiles.

The numerical ratios of the characteristic roots indicate the stiffness of the system of equations. The stiffer the system, the greater the possibility of problems in the convergence of the numerical solution. For the conditions we tested, it appears that we may not have a serious stiffness problem because the ratios of roots are approximately 1:10 to 1:25.

The results of this study also indicate that the roots do not depend much on porosity or gas velocity. There is, however, a marked dependence on liquid fraction and gas pressure.

Numerical analysis

The numerical technique chosen to solve our hyperbolic system of differential equations was the method of lines. This technique has the ability to approximate the solution of any system of linear, nonlinear or quasilinear partial differential equations that can be posed as an initial-value problem. Application of this method involves the discretization of the spacial variable and using time as the continuous independent variable. This effectively transforms the set of partial differential equations into a system of coupled ordinary differential equations with respect to time. This system can then be appropriately integrated by a sufficiently powerful ordinary differential equation integrator.

A modified version of Carver's (1974) FORSIM computer code was used as the computational tool to solve our system of equations. We upgraded FORSIM to quadruple precision. This allowed for rapid solution of equations which were previously too stiff to be solved even by integration schemes designed for stiff equations such as Gear's method. Increasing the precision allows the computer to carry more significant digits (20 for the computer system we used, Harris 800). This prevents the machine from iterating in the regime of truncation error. It has been our experience that this is important when dealing with stiff systems, since the stiffness is related to the extremes in magnitude of the terms in the equations. Normally one would expect carrying of extra significant figures to extend the computation time since more steps are required of the processor to do algebra with higher-precision numbers. But in the cases we explored, carrying more significant digits allowed for rapid convergence of the numerical scheme. The consequence of this development is not confined to our particular system of equations since FORSIM is a general-purpose software package for solving partial and ordinary differential equations. Therefore, this enhancement should prove useful in expanding the range of differential equation systems that can be resolved via the method of lines. Using our modified computer code we were able to simulate weeks of experimental time in less than 15 minutes of CPU time. Without the extra precision, the numerical scheme would use four days of CPU time and not achieve convergence to realistic solutions.

In solving the two-phase flow equations for a low-permeability core sample it has been shown that in spite of the theoretical potential to generate a mass imbalance by the separation of the terms, the nonconservative form of the governing relations are more useful in "debugging" the system (i.e., testing of boundary and initial conditions). The nonconservative form takes about half the computational time of the conservative form. Also higher-order differentiation schemes (i.e., five-point central and seven-point central) dramatically increase the computational time of the conservative form but only increase that of the nonconservative form marginally. Another advantage of the nonconservative form is that it allows for greater control over the variables and their derivatives. This can become important when investigating the effects of boundary conditions.

Gear's integration algorithm was used to solve the resultant ordinary differential equations. The Hindmarsch-Gear algorithm, although faster, has difficulty in certain regimes, i.e., higher liquid saturations. The Gear method recalculates the Jacobian matrix used to solve the algebraic relations representing the differential equations of each iteration, while the Hindmarsch-Gear only recalculates it when it cannot achieve convergence within a set tolerance after a set number of iterations.

Application of Model

To test the feasibility of the model developed, we will demonstrate its application by simulating two-phase-flow experimental studies on the IGT's experimental tight rock apparatus (ExTRA). A description of the ExTRA is given by Chowdiah (1986). The particular experiments that we consider are those in which water is drained from a partially-saturated tight sand core samples. The samples, after coming to an equilibrium condition, are exposed to a step increase in gas pressure at the core entrance. Only gas is injected at the entrance, but both liquid and gas exit the samples downstream. In contact with the exit of the core is a capillary barrier. The barrier has a capillary pressure higher than the local gas pressure thereby allowing only liquid to pass through it. This separates the two phases so that their individual flow rates can be measured. The liquid downstream of the capillary barrier is exposed to atmospheric conditions via a pipet.

The required initial and boundary conditions stem from the physical arrangement and operating conditions. Initially the sample is brought to a condition of uniform partial saturation by subjecting it to a gas pressure above that required for "gas breakthrough" for a period of time sufficient to allow the system to come to equilibrium. The gas pressure needs to be greater than or equal to the breakthrough pressure to ensure that the gas phase is continuous across the sample. This equilibrium state is physically achieved when the liquid stops draining out of the sample. At that time, we will assume that the liquid pressure is uniform and equal to the reference pressure, to which the liquid is exposed via the pipet.

Since we have assumed that the system has come to equilibrium with a uniform gas density across the core, ρ_{go}, we can determine the initial liquid volume fraction of the system. Taking the liquid pressure as equal to its reference pressure, P_{lo}, and using the system gas pressure, we can obtain the initial capillary pressure from the constraint of capillary equilibrium:

$$P_c^0 = P_g^0 - P_{lo} \tag{21}$$

If we assume that the capillary pressure is a predetermined function of fluid volume fraction, we can obtain a relation for the starting liquid volume fraction, S_{lo}:

$$f(S_l^0) = P_g^0 - P_{lo} \tag{22}$$

The fluid velocities are initially zero by our equilibrium assumption.

The initial conditions then become:

$$\begin{aligned} A &= \phi(l - S_{lo} - S_{gl})\, \rho_{go} \\ B &= 0.0 \\ E &= \phi(S_{lo} - S_{ll})\, \rho_{lo} \\ F &= 0.0 \end{aligned} \tag{23}$$

where ρ_{lo} is the liquid density at the reference pressure P.

At time "zero plus," the upstream end of the core is exposed to a step increase in gas pressure. The gas pressure at the exit is maintained at its initial condition. No liquid enters the sample; therefore, we have assumed that the liquid velocity is zero at the upstream end of the sample. This gives us three of the four boundary conditions necessary to define the system.

The last boundary condition is not as straightforward. Since the capillary barrier allows only liquid to pass through it, we have applied Darcy's law for incompressible single-phase flow-through the barrier. We also assumed that the liquid leaving the capillary barrier of thickness, Δx, is exposed to the reference pressure, P_{lo}, and that the volumetric flow rate of liquid leaving the core sample equals the flow through the totally-saturated barrier. Then, the following relation can be developed for the liquid velocity at the core exit.

$$U_l = \frac{[P_g - P_{lo} - f(S_l)]\, k_{lB}}{\mu_l\, \phi(S_l - S_{ll})\, \Delta x} \tag{24}$$

where K_{lB} is the liquid permeability in the barrier.

In terms of our equation variables, the upstream boundary conditions become:

$$\begin{aligned} A &= \phi(l - S_l - S_{gl})\rho_g^* \\ F &= 0.0 \end{aligned} \tag{25}$$

where the gas density ρ_g^*, corresponds to the step increase in gas pressure at the sample entrance. At the sample exit, the boundary conditions are:

$$\begin{aligned} A &= \phi(l - S_l - S_{gl})\rho_g^0 \\ F &= \frac{\rho_l[P_g^0 - P_{lo} - f(S_l)]\, k_{lB}}{\mu_l\, \Delta x} \end{aligned} \tag{26}$$

The rock properties used in this study are those experimentally obtained for Plug 6686 at IGT (GRI Final Report 1986). The specific operating conditions correspond to steady-state gas flow rates on the order of 10^{-3} std. cm^3/s and exit liquid flow rates of the order of 10^{-7} cm^3/s.

The ideal gas law modified by a compressibility factor was used to represent the gas density. The compressibility factor was assumed to be a constant, since the system is isothermal and the pressure drop across the sample is very small.

The liquid density is taken as a constant equal to its initial atmospheric pressure value. This assumption is appropriate since the pressure change in the liquid phase from atmospheric conditions will be small (less than or equal to the gas pressure drop). Table 1 shows the input data for plug number 6686 with plug length of 3.88 cm and cross-sectional area of 5.05 cm^2. The nonwetting fluid was nitrogen and wetting fluid was water.

Both the capillary pressure expression and gas permeability expression were obtained using experimental data conducted on Plug 6686. Since no data was available for liquid permeability, we assumed a form of Corey's Relation (1954).

Figure 2 shows the calculated gas pressure distribution in the core as a result of the step increase in entrance pressure from the initial equilibrium condition of 865.0 kPa to 935.0 kPa. The ordinate is made dimensionless by subtracting the downstream gas pressure, P_{go}, from the gas pressure at a position, P_g, and dividing by the overall gas pressure drop across the sample ($P_g - P_{go}$). Initially, the relative gas pressure across the sample is zero. After the step increase has occurred, the relative gas pressure at the entrance is one. The figure shows that this pressure wave is quickly felt throughout the sample. A point of interest in this figure is that the steady-state gas pressure profile is not linear. It is above and concave to a linear distribution from 1 to 0 across the sample. The linear distribution is what one would expect for the steady-state condition of single-phase gas flow.

Table 1. Input Data for Plug Number 6686

Properties	Numerical Value
Dry Porosity	0.1367
Liquid Permeability of Capillary Barrier	7.896 μm^2
Irreducible Wetting Fluid Saturation, S_{ll}	0.4065
Irreducible Gas Saturation, S_{gl}	0.0
Mean Gas Pressure	900.0 kPa
Gas Pressure Drop	70.0 kPa
Temperature	306.5 K
Liquid Density	0.9947 gm/cm^3
Liquid Viscosity	0.744 mPa·S
Capillary Pressure	$P_c = 71.0 + 306.1/S_l^2 - P_{lo}$
Gas Permeability	$k_g = 31.9 \times 10^{-6} (1.0 + 296.0/P_g)$ $[-0.0995 - 0.472 \ln(S_l)]$
Liquid Permeability	$k_l = 10.02 \times 10^{-6}$ $[(S_l - S_{lo})]/(1 - S_{lo})]^4$

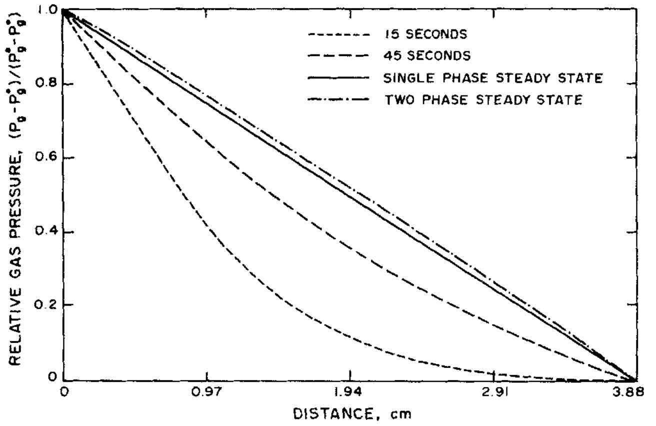

Figure 2. Gas pressure distribution in a low-permeability core sample at constant pressure drop.

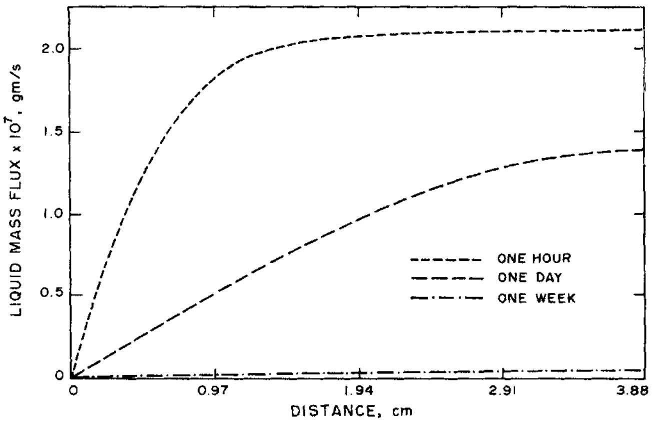

Figure 4. Liquid mass flux distribution across a low-permeability core.

The nonlinear gas pressure profile, as steady state is approached, is a consequence of the capillary equilibrium imposed on the system. The capillary pressure expression relating to the fluid pressures is nonlinear. The liquid flows from the high-pressure end to the low-pressure end of the sample. The liquid volume fraction becomes higher as the sample exit is approached. From the gas permeability relation, the higher the liquid saturation, the greater the resistance to gas flow. This means that for a constant gas flow rate through the sample, the magnitude of the gas pressure drop increases as we move towards the exit of the sample. Therefore, at a steady-state condition, when the liquid phase is stationary and the gas mass flux is constant throughout the sample, the gas pressure profile should be concave with respect to the origin.

Figure 3 displays the gas mass flux profile referenced to its steady-state value at various times. The gas mass flux approached its steady-state value after 90 seconds. However, true steady state cannot be achieved until the liquid finishes migrating in the sample. Figure 4 shows how the liquid mass flux varies across the sample at various times. Since no liquid is allowed to enter the sample, the liquid flux at the entrance is zero. The steady-state condition for the liquid phase is a condition of zero mass flux. For this case, it takes over a week for the liquid to approach steady state. The liquid may be moving a thousand times or more slower than the gas phase. Therefore, a true steady state will take much longer than for a "pseudosteady state." For convenience, we will define the "pseudosteady state" as the condition for which the gas phase is only waiting for the liquid phase to reach its steady-state value so that a true steady state can be achieved.

Figure 5 shows how the liquid volume fraction varies across the sample as a function of time. Again, it is obvious that the liquid phase takes much longer to approach its steady state. It takes weeks for the liquid to stop draining from the sample.

The exit gas velocity reaches its pseudosteady value of approximately 93% of its true steady-state value in a few minutes. It continues to increase very gradually as the liquid slowly drains out of the sample and thereby increases the gas permeability.

Figure 6 shows the exit liquid flow rate over seven days of operation. Since no liquid is allowed to enter the sample (drainage experiment), the maximum liquid flow rate occurs shortly after the step increase of inlet gas pressure. As time proceeds, the flow of liquid gradually decreases and becomes zero. When compared to the steady-state gas mass flux, the mass flux of liquid is approximately 1,000 times smaller. Or in terms of volumetric flow rate at standard conditions, the gas moves 10,000 times faster than the liquid.

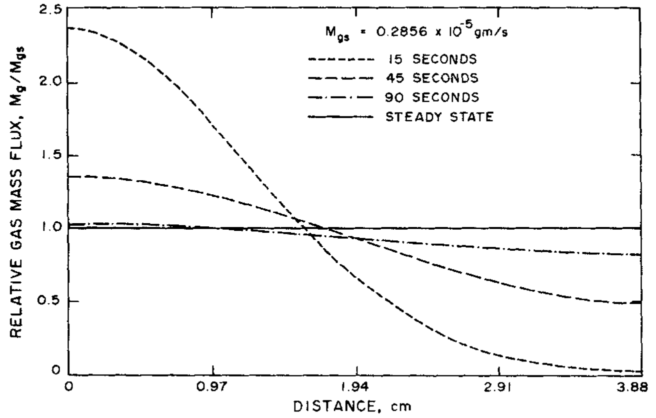

Figure 3. Gas mass flux distribution across a low-permeability core.

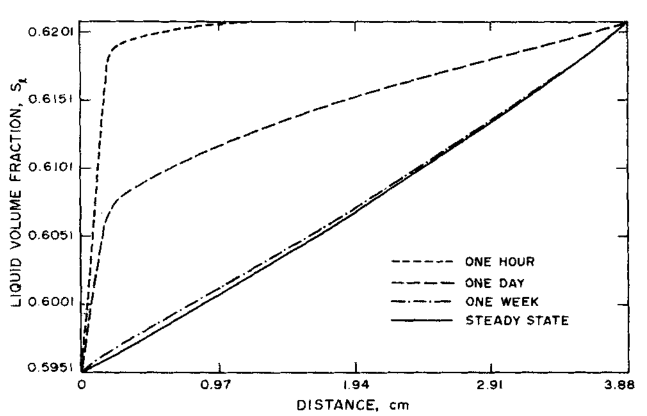

Figure 5. Liquid saturation distribution across a low-permeability core.

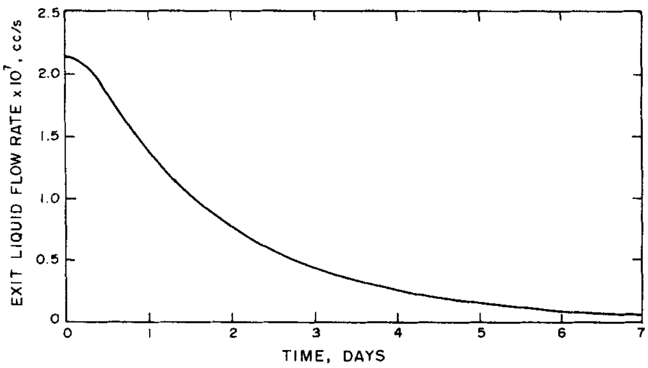

Figure 6. Exit liquid flow rate from a low-permeability core during one week of drainage experiment.

Our Model Prediction vs. Darcy's Law and Experimental Data

The lack of complete and reliable two-phase-flow experimental data did not permit us to compare our calculated values with two-phase flow data. Instead, we have compared our calculated values for exit gas flow rate through partially-saturated low-permeability media (the liquid does not flow out of the core sample) with IGT's experimental data (Randolph, 1983; Chowdiah and Arastoopour, 1983). Furthermore, we compared exit gas flow rate predicted by our model with calculated values using Darcy's Law as momentum equation (neglecting momentum outflow and momentum accumulation terms in our model). Figure 7 shows the comparison between exit flow rate calculated from our model with Darcy's Law's prediction and IGT's experimental data. The exit gas flow rate calculated using Darcy's Law deviates considerably during the transient period of the experiment from calculated values using our model and IGT's experimental data. Since for low-permeability media the transient data are being used to obtain flow parameters (steady state reaches after a long time), better agreement between our model and experimental data is very important. Our model described the data well during the transient period. At the steady-state conditions, both models predict the experimental data well. The Plug Number 6547 was used for experimental data in Figure 7. This plug has 22.6% water saturation and porosity and permeability of 0.0223 and 0.475 × 10^{-6} μm^2, respectively.

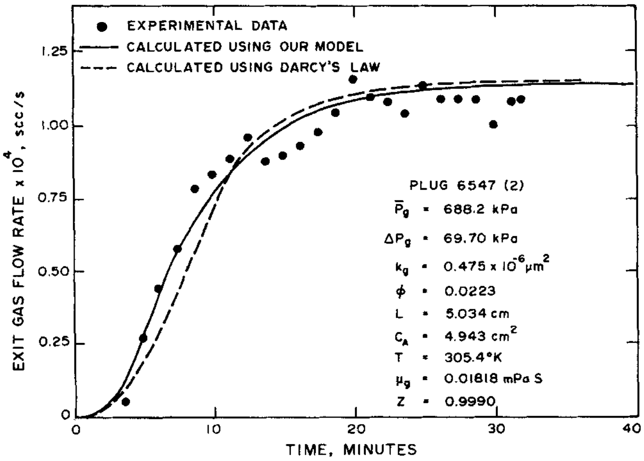

Figure 7. Comparison of our model with Darcy's Law prediction and gas exit flow data for plug 6547(2) with 22.6% water saturation.

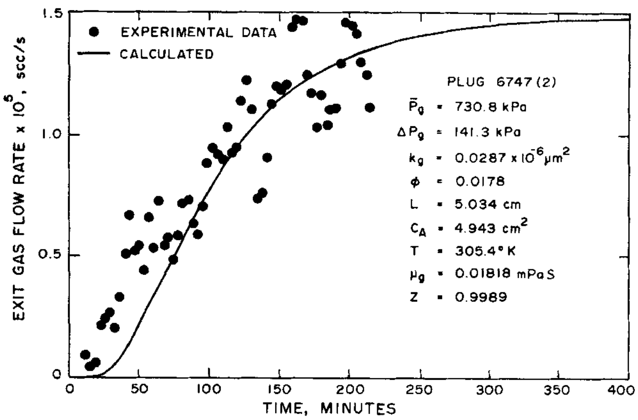

Figure 8. Comparison of our model prediction with gas exit flow data for plug 6547(2) with 38.2% water saturation.

Figure 8 shows the comparison of exit gas flow rate calculated from our model with IGT's experimental data for Plug Number 6547 with 38.2% water saturation and porosity and permeability of 0.0178 and 0.0287 × 10^{-6} μm^2. Our calculated values agreed with experimental data reasonably well. The slight data fluctuation and deviation from smooth calculated gas flow rate are due to temperature fluctuation during the experiments. These favorable comparisons are the first step indicating the potential of our model and numerical technique in describing gas, and liquid flow through low-permeability porous media. Complete experimental data for flow of gas and liquid through low permeability media are needed to test the full capability of our model.

Conclusion

A transient one-dimensional isothermal two-phase flow model has been developed for the flow of gas and liquid through a low-permeability porous medium using the general continuity and momentum equations. The viscous force terms in the momentum relations are expressed as functions of phase permeabilities such that with further assumptions these equations can be reduced to Darcy's Law. The fundamental assumptions made in the development of the model are that the momentum transfer between fluids inside the individual pores is negligible, and a condition of capillary equilibrium exists throughout the medium.

The system of governing differential equations has been demonstrated to be well posed as an initial value problem under the reservoir conditions. The characteristics of the equations are all real and distinct. The solutions of the simulation appear to exist and be unique.

The differential equations have been solved for single-phase as well as two-phase flow conditions. These conditions were previously too stiff to solve numerically via the *method of lines*. We accomplished this through the modification of the computer code.

Our calculated flow parameters showed that there is a dramatic difference in time required for the gas phase to achieve an apparent steady and for the system to approach true steady state. Imposing capillary equilibrium upon the system accounts for the degree of nonlinearity of both steady-state pressures and saturation profiles. Capillary pressure relates fluid pressures inside the medium and quantifies the driving force for the migration of liquid in the medium. It also accounts for the accumulation of liquid at the low-pressure end of the sample.

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Notation

C = liquid compressibility factor
C_A = cross-sectional area of core sample
F_{ij} = interfacial drag between phase i and j
k = permeability
L = length
M = mass flow rate
MW = gas molecular weight
P = pressure
P_c = capillary pressure
Q = volumetric flow rate
R = universal gas constant
S = volume fraction
T = absolute temperature
t = time
U = local averaged pore velocity
x = distance
z = gas compressibility factor

Greek letters

γ = volume porosity
Δ = difference
μ = viscosity
ρ = density
σ = specific free energy of an interface
ϕ = flow porosity

Subscripts

B = capillary barrier
g = gas phase
I = immobile phase
i = irreducible
l = liquid phase
M = mobile phase
nw = nonwetting phase
o = reference condition
s = steady state
w = wetting phase

Superscripts

$*$ = imposed boundary condition
o = imposed initial condition

Literature Cited

Arastoopour, H., and M. Adewumi, "Single and Two Phase Flow Equations in Low-Permeability Porous Media," Multiphase Flow and Heat Transfer Symposium Workshop, Miami Beach (Apr. 18–20, 1983).

Chowdiah, P., "Effects of Pore Water Distribution and Stress on the Laboratory Measurement of Tight Sandstone Properties," paper 15210, SPE Unconventional Gas Technol. Symp., Louisville, KY (May, 1986).

Chowdiah, P., and H. Arastoopour. "Measurement and Analysis of Transient Flow of Gas Through Tight Sand Core Samples," *Proc. Intersoc. Energy Conversion Eng. Conf.*, 480 (1983).

Carver, M. B., "FORSIM: A FORTRAN Package for the Automated Solution of Coupled Partial and/or Ordinary Differential Equation Systems," Chalk River Nuclear Laboratories, Chalk River, Ont. (1974).

Collins, R. E., and C. E. Cooke, "Fundamental Basis for the Contact Angle and Capillary Pressure," *Trans. Faraday Soc.*, **55**, 1602 (1959).

Corey, A. T., "The Interrelation Between Gas and Oil Relative Permeabilities," *Producers Monthly*, 38 (Nov., 1954).

Freeman, D. L., and B. C. Bush, "Low Permeability Laboratory Measurements by Nonsteady-State and Conventional Methods," *SPE J.*, **23**, 928 (1983).

Gidaspow, D., C. Seo, and B. Ettehadieh, "Hydrodynamics of Fluidization Experimental and Theoretical Bubble Size In A Two-Dimensional Bed with A Jet," *Chem. Eng. Commun.*, **22**, 253 (1983).

Glasstone, S., *Textbook of Physical Chemistry*. 2nd ed., MacMillan, London (1962).

Gray, W. G., and K. O'Neil, "On the General Equations for Flow in Porous Media and Their Reduction to Darcy's Law," *Water Resources Res.*, **12**(2), 148 (1976).

GRI Final Report, "Measurement and Analysis of Two-Phase Flow Through Low Permeability Media," **1** and **2**, submitted to GRI by IGT, Contract No. 5082-260-0711, available from NTIS (Nov. 1982–Mar. 1986).

Jones, F. O., and W. W. Owens, "A Laboratory Study of Low Permeability Gas Sands," *J. Pet. Tech.*, 1631 (1980).

Lax, P. D., "Differential Equations, Difference Equation and Matrix Theory," *Commun. Appl. Math*, **11**, 175 (1958).

Lyczkowski, R. W., D. Gidaspow, C. W. Solbring, and E. D. Hughes, "Characteristics and Stability Analysis of Transient One-Dimensional Two Phase Flow Equations and Their Finite Difference Approximation," *Nucl. Sci. and Eng.*, **66**, 378 (1978).

Newberg, M. A., and H. Arastoopour, "Analysis of the Flow of Gas Through Low Permeability Porous Media," *SPE Reservoir Eng.*, 647 (1986).

Philip, J. R., "Flows Satisfying Mixed No-Slip and No-Shear Conditions," *J. Appl. Math., Phys. (ZAMP)*, **23**, 353 (1972).

Randolph, P. L., "Porosity and Permeability of Mesa Verde Sandstone Core from the U.S. DOE Mulliwell Experiment, Garfied County, Colorado," *Proc. SPE/DOE Joint Symp. on Low Permeability Gas Reservoirs*, 449 (1983).

Rapoport, L. A., and W. J. Leas, "Relative Permeability to Liquid in Liquid-Gas Systems," *AIME Pet. Trans.*, **192**, 83 (1951).

Rietma, K., and S. M. P. Mutsers, "The Effect of Interparticle Force on Expansion of a Homogeneous Gas-Fluidized Bed," *Proc. Int. Conf. on Fluidization*, 32, Toulouse, France (1973).

Semrau, J., "Analysis of Two Phase Flow through Low-Permeability Media," PhD Thesis, Illinois Institute of Technology (1986).

Sharer, J. C., and P. O'Shea, "GRI's Research Program on Unconventional Natural Gas," *Chem. Eng. Prog.*, 19 (1986).

Slattery, J. C., "Flow of Visoelastic Fluids through Porous Media," *AIChE J.*, **13**(6), 1066 (1967).

Slattery, J. C., "Two-Phase Flow Through Porous Media," *AIChE J.*, **16**(3), (1970).

Soeder, D. J., "Laboratory Drying Procedure and Permeability of Tight Sandstone Core," *SPE Formation Evaluation*, 16 (1986).

Whitaker, S., "Flow in Porous Media: I & II," AIChE Meeting, Chicago (Nov., 1985).

———, "The Transport Equations for Multi-Phase System," *Chem. Eng. Sci.*, **28**, 139 (1973).

Wooding, R. A., and H. J. Morel-Seytoux, "Multiphase Fluid Flow Through Porous Media," *Ann. Rev. Fluid Mech.*, **8**, 233 (1976).

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